A Reply to a Comment on "Dark matter: A phenomenological existence proof"

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Abstract. In astro-ph/0601489, within the framework of the Einsteinian general relativity, we made the observation that if the universe is described by a spatially flat Friedmann-Robertson-Walker (FRW) cosmology with Einsteinian cosmological constant then the resulting cosmology predicts a significant dark matter component in the universe. Furthermore, the phenomenologically motivated existence proof refrained from invoking the data on galactic rotational curves and gravitational lensing, but used as input the age of the universe as deciphered from the studies on globular clusters. This claim has been challenged in astro-ph/0603213. Here we show that the raised objection is invalid. It, at best, constitutes a trivial consistency check. As such, we stand by our analysis, and by our conclusions, without reservations.

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Referring the reader to Ref. [1] for notational details we recall that there we considered a FRW cosmology defined by the set

$$\{k=0,\ w^{\Lambda}=-1,\ \rho=\rho_{\mathrm{m}},\ p=p_{\mathrm{m}}=0,\ \ \rho^{\Lambda}=\text{constant}\} \tag{1}$$

and proved, without further assumptions, that

$$\Omega_{\mathrm{m}}(t):\Omega_{\Lambda}(t)=1:\zeta(t) \tag{2}$$

where $\zeta(t) := \sinh^2\left(\sqrt{3}\,t/(2\tau_{\Lambda})\right)$, and we defined $\tau_{\Lambda} := \sqrt{1/\Lambda}$. We took the observational value of Λ, defined a reference unit of time, and explicitly stated it — something which the author of Ref. [2] seems to have ignored in making his objections.

To avoid semantical misinterpretation we note that we consider a cosmology with Einsteinian cosmological constant. To say that this cosmology, in any sense, does not invoke a specific value of Λ carries no content (and is wrong). This statement holds true in the numerical sense, rather than analytical. For the latter, no specific value of Λ need be invoked.

Now $\zeta(t) := \Omega_{\Lambda}/\Omega_{\mathrm{m}}$ is a *unique* function of t — or, if one wishes to be pedantic, of t/τ_{Λ}.

Once the age of the universe is specified by some *independent* observations,‡ the considered cosmology uniquely determines the ratio $\Omega_{\mathrm{m}}(t) : \Omega_{\Lambda}(t)$. A graphical representation of $\zeta(t)$ is given in [1, Fig. 1]. In addition, the fractional matter density is *predicted*

$$\Omega_{\mathrm{m}}(t)=(1+\zeta(t))^{-1}\,. \tag{3}$$

This is the combined result of equation (2) and of fact that we are considering a spatially flat cosmology, which requires $1=\Omega_{\mathrm{m}}+\Omega_{\Lambda}$. A graphical representation of $\Omega_{\mathrm{m}}(t)$ is given in [1, Fig. 2].

In Ref. [1] we emphasized that this circumstance arises due to a specific nonlinear aspect of Einstein's field equations for the considered FRW cosmology. Author of Ref. [2] has failed to appreciate its impact and importance in the argument we presented.

We chose for t the age of the universe as deciphered from the age of the globular clusters and arrived at the claims contained in Ref. [1]. Briefly, we fixed a range of t as indicated;

‡ We shall not take issue even if this assertion was to be totally abandoned. To do so would be against the spirit and content of our argument contained in Ref. [1].

that yielded a range of ζ *and* $\Omega_{\rm m}$ (a fact completely missed by the author of Ref. [2]). The latter Ω is the sum of all non-relativistic matter components. Taking the standard model contribution to $\Omega_{\rm m}$ as 0.05, we deciphered that there must exist an additional (dark matter, by definition) contribution to $\Omega_{\rm m}$ in the rough range $0.14 \leq \Omega_{\rm dm} \leq 0.30$.

We did not invoke as input any value of ζ, contrary to the assertion of Ref. [2]. Once the range of t was fixed, it *automatically* constrained ζ to $1.9 \leq \zeta \leq 4.3$; that is, the ratio $\Omega_\Lambda/\Omega_{\rm m}$.

Finally, in order to avoid confusion we note

- We have no disagreement with the remarks made by the author of Ref. [2] regarding the age of the universe and the age of the globular clusters. We did not use the age of the globular clusters but the age of the universe as deciphered from that age. However, even if one equated the two no significantly different conclusion is arrived at (as long as one does not invoke 'alternate cosmologies').
- The quoted value of $\Omega_\Lambda = 0.73$ invoked in Ref. [2] was *not* assumed *a priori* (by us), but it follows once the age of the universe is specified. This is manifest from a glance at Fig. 1 of Ref. [1]. However, if one chooses to specify the age of universe using a specific value of Λ, or by some other means, one obtains a simple consistency check; and indeed that in essence is the core result of Ref. [2]. To be precise, all interpretations of the age of the universe are specific to cosmological models. But such a discussion, while of much physical significance, is not the primary task of the discussion at hand.
- In general the $\zeta(t)$, i.e. the ratio $\Omega_\Lambda/\Omega_{\rm m}$, is not fixed for a given epoch within the context of general relativistic cosmological models. What usually happens is that *given a priori* specific initial value for $\Omega_\Lambda/\Omega_{\rm m}$ its temporal evolution is predicted. For the model defined by (1) the situation is a bit more subtle [1] due to a nonlinear aspect of Einstein's field equations for the considered FRW cosmology. This is fully explained in Ref. [1] and its full appreciation is necessary to reach the results contained therein.

As such we stand by our conclusion that if the universe is described by a spatially flat Friedmann-Robertson-Walker (FRW) cosmology with Einsteinian cosmological constant then the resulting cosmology predicts a significant dark matter component in the universe. Furthermore, the phenomenologically motivated existence proof refrains from invoking the data on galactic rotational curves and gravitational lensing, but uses as input (a) the age of the universe as deciphered from studies on globular clusters, and (b) $\Omega_{\rm sm} \approx 0.05$, where the subscript stands for 'standard model' component.

References

[1] D. V. Ahluwalia-Khalilova, Dark matter: A phenomenological existence proof, arXiv:astro-ph/0601489.

[2] R. G. Vishwakarma, Comments on "Dark matter: A phenomenological existence proof," arXiV:astro-ph/0603213.